

# Waiting lines formulas, KPP227

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## Waiting lines

### Interarrival times

$$P_n = \frac{(\lambda T)^n}{n!} e^{-\lambda T} \text{ for } n = 1, 2, \dots$$

$P_n$  = Probability of  $n$  arrivals in  $T$  time periods

$\lambda$  = Average numbers of customer arrivals per period

### Service time distribution

$$P_{(t \leq T)} = 1 - e^{-\mu T}$$

$\mu$  = Average number of customers completing service per period

$t$  = service time of the customer

$T$  = target service time

## Single server model

Average utilization of the system

$$\rho = \frac{\lambda}{\mu}$$

Average number of customers in the system

$$L = \frac{\lambda}{\mu - \lambda}$$

Average number of customers in the waiting line

$$L_q = \rho L = \frac{\lambda^2}{\mu(\mu - \lambda)}$$

Average time spent in the system including service

$$W = \frac{1}{\mu - \lambda}$$

Average waiting time in line

$$W_q = \rho W = \frac{\lambda}{\mu(\mu - \lambda)}$$

Probability that  $n$  customers are in the system

$$P_n = (1 - \rho)\rho^n$$

Probability that 0 customers are in the system

$$P_0 = 1 - \frac{\lambda}{\mu}$$

Probability that less than  $k$  customers are in the system

$$P_{n < k} = 1 - \left(\frac{\lambda}{\mu}\right)^k$$

Probability of more than  $k$  customers are in the system

$$P_{n > k} = \left(\frac{\lambda}{\mu}\right)^{k+1}$$