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Forecasting

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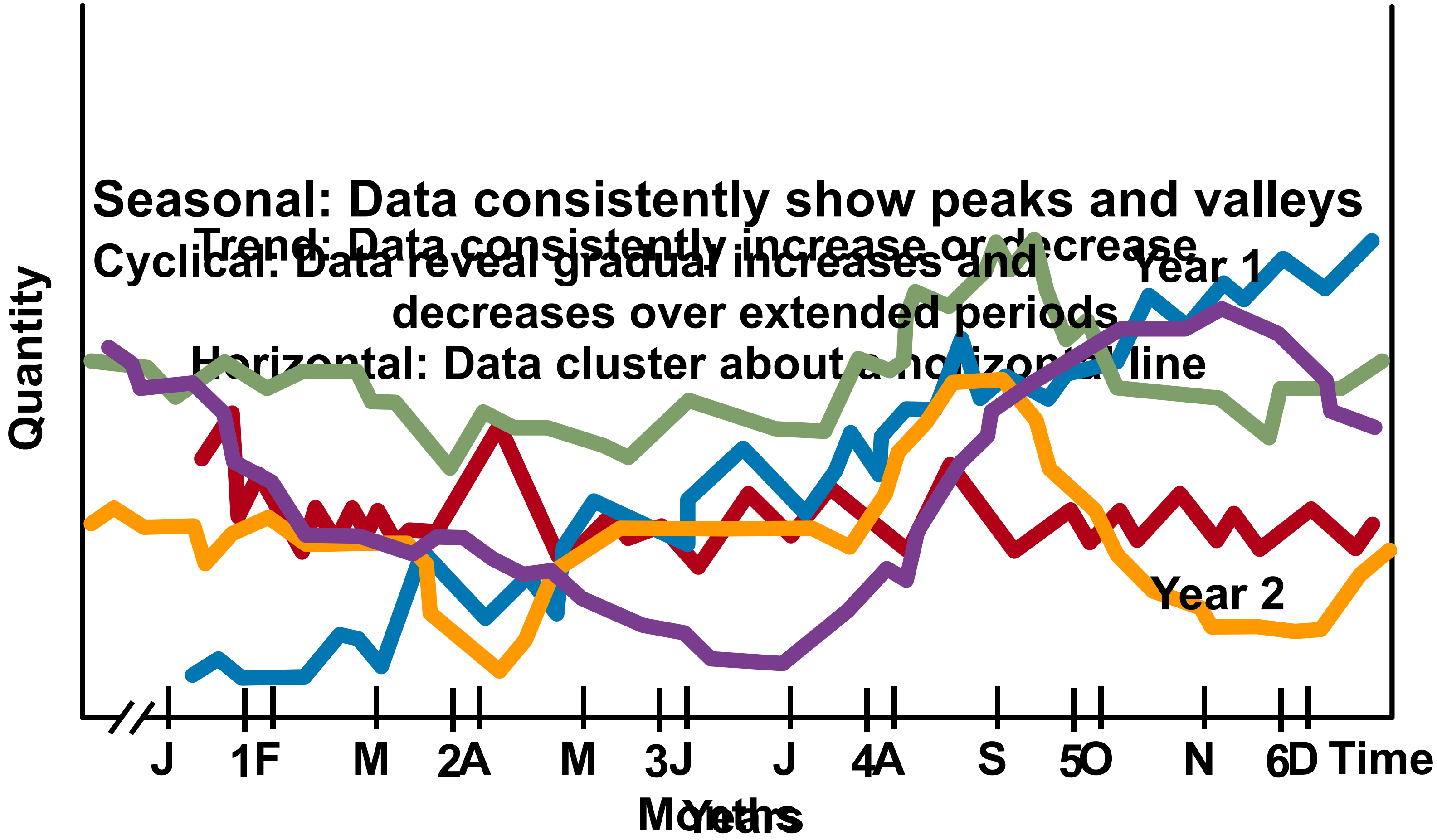
Forecasting

- Forecasts are critical inputs to business plans, annual plans, and budgets
- Finance, human resources, marketing, operations, and supply chain managers need forecasts to plan: output levels, purchases of services and materials, workforce and output schedules, inventories, and long-term capacities
- Forecasts are made on many different variables
- Forecasts are important to managing both processes and managing supply chains

Forecasting

- Forecasts assume causal systems (past → future)
- Forecasts are rarely perfect because of randomness
- Forecasts are more accurate for groups than for individuals
- Forecast accuracy decreases as time horizon increases

Demand patterns



Forecasting techniques

Judgement methods:

When lacking historical data, firms rely on managerial judgment and experience to generate forecasts. Methods used are:

- Sales force estimates
- Executive opinion
- Market research
- Delphi method

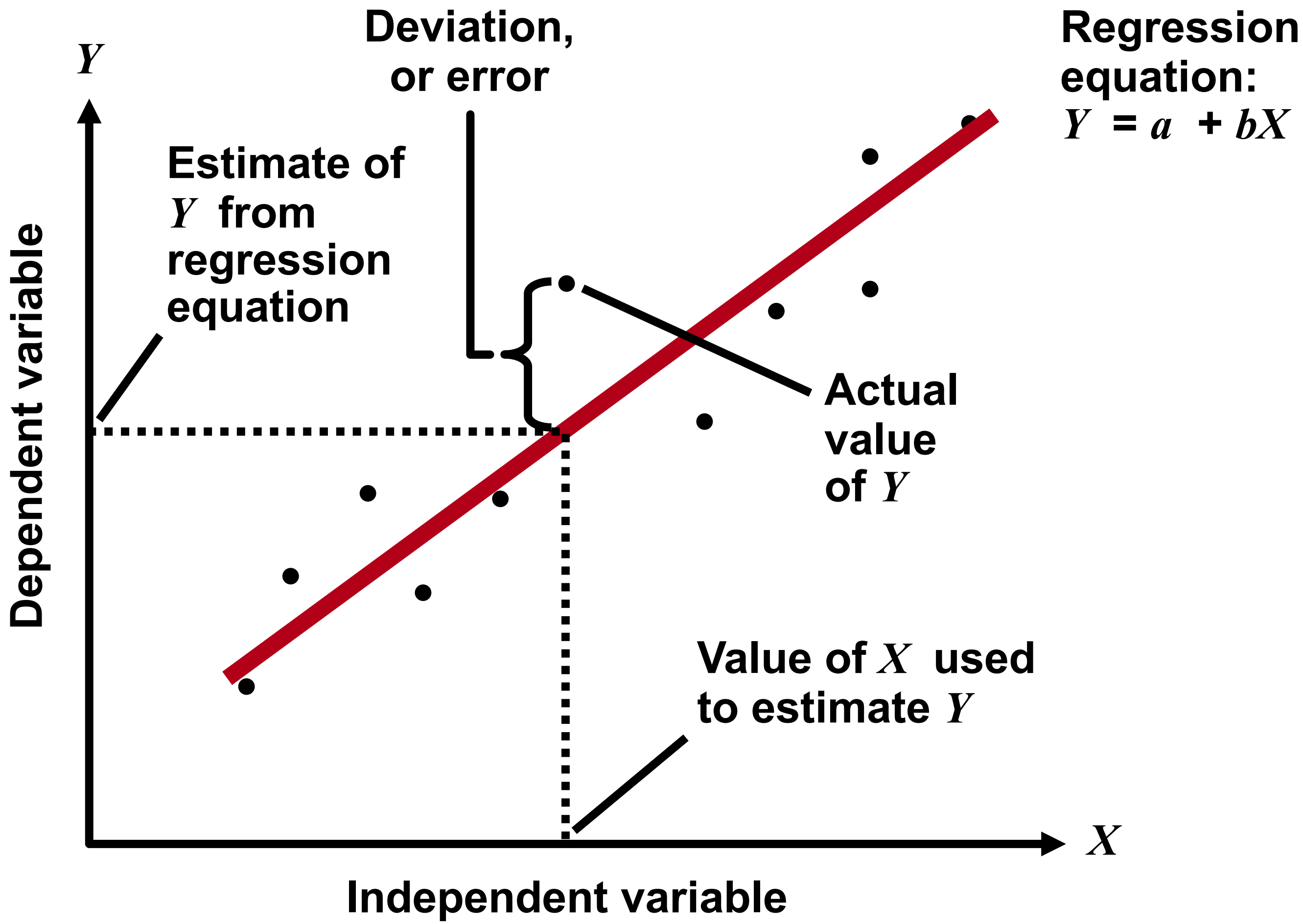
Forecasting techniques

Causal methods:

When historical data is available and the relationship between the factor to be forecasted and other external or internal factors can be identified.

Linear regression is the most commonly used causal method.

Linear regression



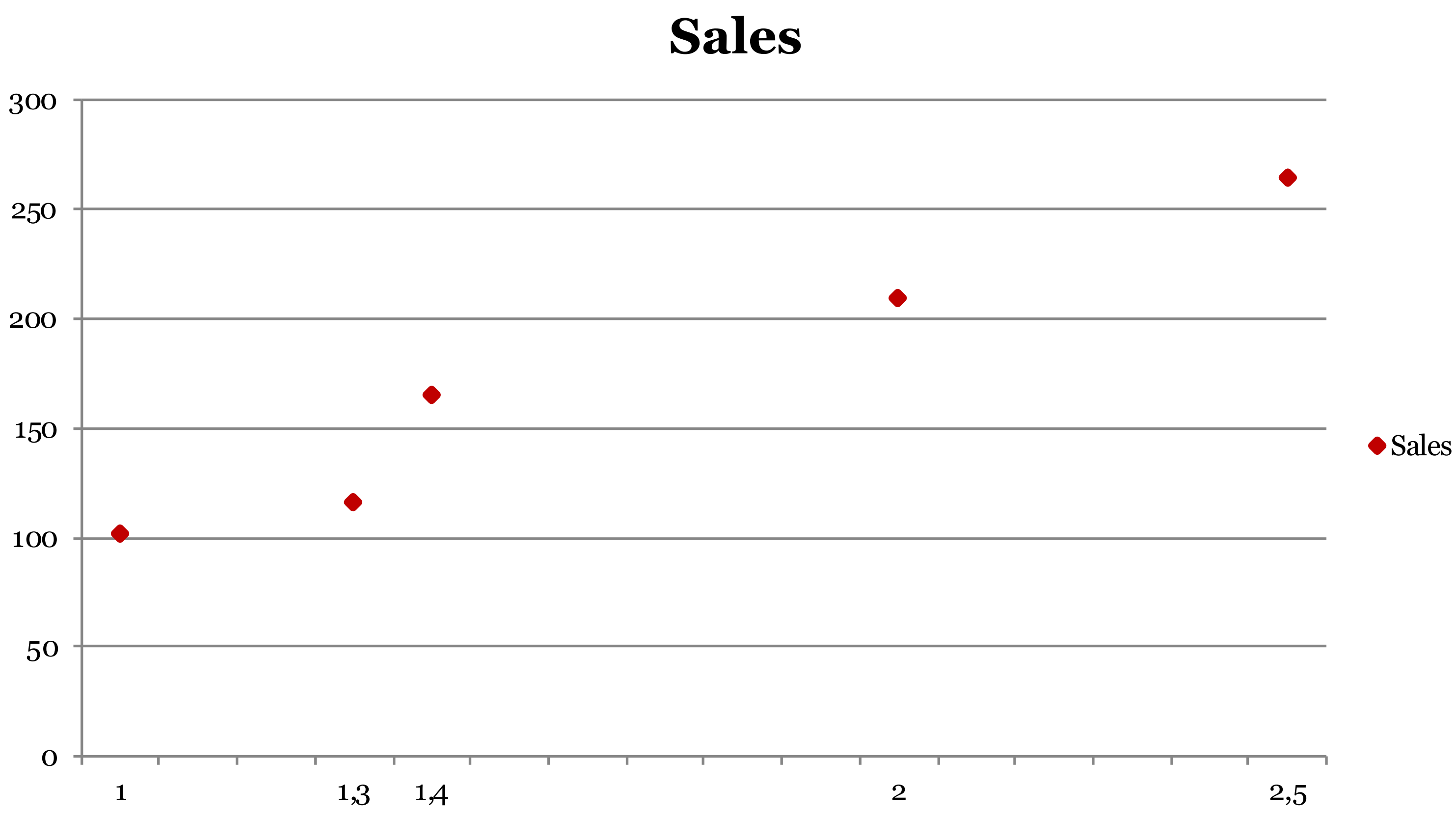
Linear regression

In order to plan for our production, we have to prepare a forecast of our product demand. To guide us, the marketing manager has provided us with the advertising budget for a brass door hinge.

Month	Sales (thousands of units)	Advertising (thousands of €)
1	264	2,5
2	116	1,3
3	165	1,4
4	101	1
5	209	2

The marketing manager says that next month, the company will spend 1750 € on advertising for the product. Use linear regression to develop an equation and a forecast for this product.

Linear regression



Linear regression

Month	Advertising X	Sales Y	XY	X^2	Y^2
1	2,5	264	660	6,25	69696
2	1,3	116	150,8	1,69	13456
3	1,4	165	231	1,96	27225
4	1	101	101	1	10201
5	2	209	418	4	43681
Total:	8,2	855	1560,8	14,9	164259

$$\bar{X} = \frac{8.2}{5} = 1.64 \qquad \bar{Y} = \frac{855}{5} = 171.00$$

Linear regression

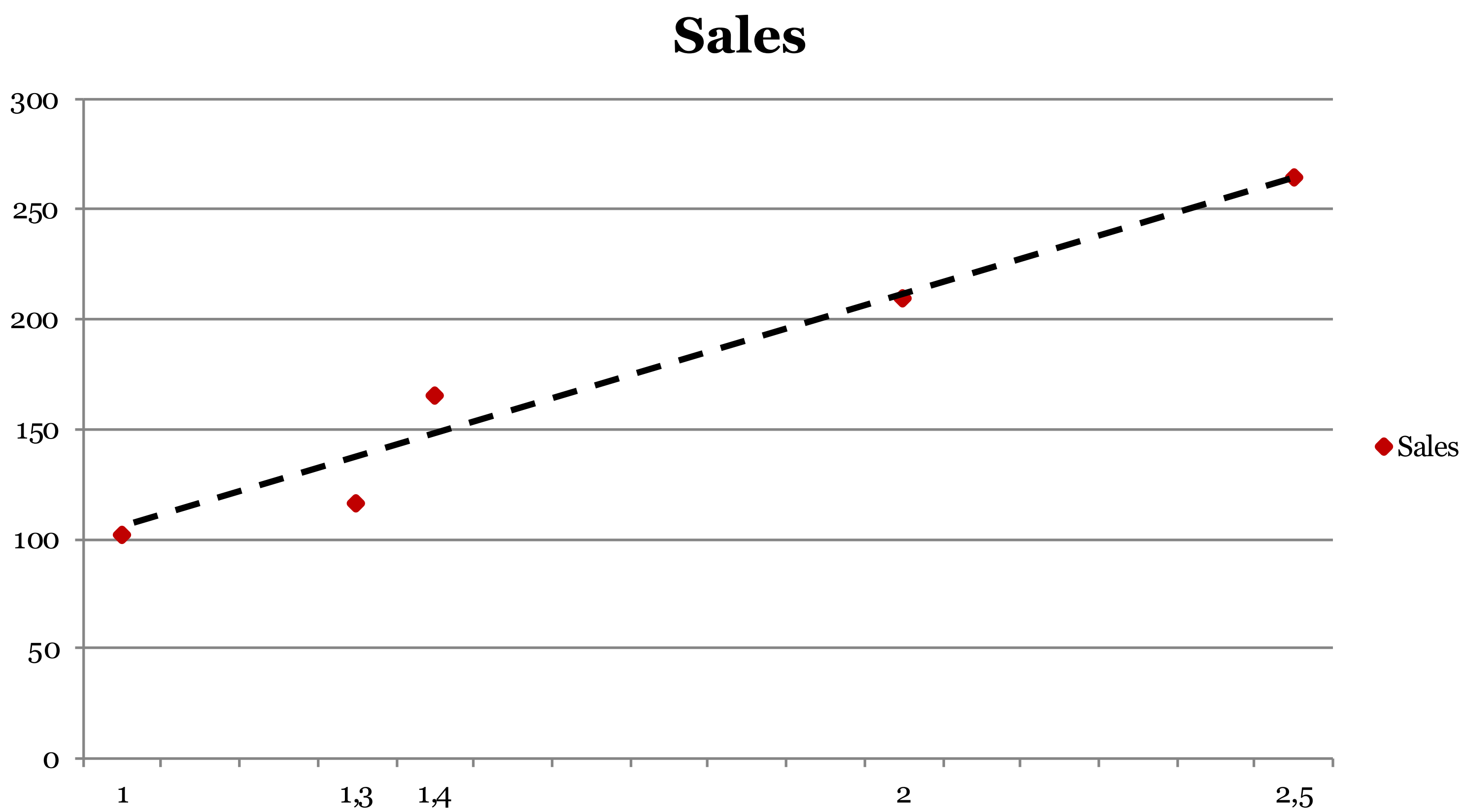
Regression equation: $Y = a + bX$

$$b = \frac{\sum XY - n\bar{X}\bar{Y}}{\sum x^2 - n\bar{X}^2} = \frac{1560.8 - 5(1.64)(171)}{14.9 - 5(1.64)^2} = 109.229$$

$$a = \bar{Y} - b\bar{X} = 171.00 - 109.229(1.64) = -8.136$$

$$Y = -8.136 + 109.229X$$

Linear regression



Linear regression

Forecast accuracy

Correlation coefficient

$$r = \frac{n\sum XY - \sum X \sum Y}{\sqrt{[n\sum X^2 - (\sum X)^2][n\sum Y^2 - (\sum Y)^2]}}$$

Value: -1 to +1
0 = No correlation
-1 = negative correlation
+1 = positive correlation

Coefficient of determination

$$r^2 = \frac{a\sum Y + b\sum XY - n\bar{Y}^2}{\sum Y^2 - n\bar{Y}^2}$$

Value: 0-1
Shows to what extent the variations in Y is explained by the x-values

Standard error of the estimate

$$\sigma YX = \sqrt{\frac{\sum Y^2 - a\sum Y - b\sum XY}{n-2}}$$

Indicates the average deviation between the regression line and the true outcome

Linear regression

Forecast accuracy

$$r = \frac{5(1560.8) - (8.2)(855)}{\sqrt{[5(14.90) - (8.2)^2][5(164259) - (855)^2]}} = 0.98$$

$$r^2 = \frac{-8.136(855) + 109.229(1560.8) - 5(171)^2}{164259 - 5(171)^2} = 0.96$$

$$\sigma YX = \sqrt{\frac{164259 - (-8.136)(855) - 109.229(1560.8)}{5 - 2}} = 15.61$$

Linear regression

$$Y = -8.136 + 109.229X$$

$$Y = -8.136 + 109.229(1.75) = 183.015$$

The forecast for month 6 is: 183015 units.

Linear regression

In our factory, a machine cell is cutting gears. The normal times (in minutes) for cutting an eight-inch-diameter gear for the last five jobs are shown in the following table. The next eight-inch-diameter gear is to have 20 teeth. Estimate how long cutting the gear will take

Job number	Cutting time	Number of teeth
2542	115	23
2557	84	17
2571	52	10
2593	138	28
2611	67	14

Linear regression

Job no.	Teeth, X	Cutting time, Y	XY	X^2
2542	23	115	2645	529
2557	17	84	1428	289
2571	10	52	520	100
2593	28	138	3864	784
2611	14	67	938	196
Total:	92	456	9395	1898

$$\bar{X} = \frac{92}{5} = 18.4 \qquad \bar{Y} = \frac{456}{5} = 91.2$$

Linear regression

$$b = \frac{9395 - 5(18.4)(91.2)}{1898 - 5(18.4)^2} = 4.896$$

$$a = 91.2 - 4.896(18.4) = 1.1136$$

$$Y = 1.1136 + 4.896(20) = 99.034$$

The estimated time for cutting the 20 teeth gear is: 99.034 minutes.

Forecasting techniques

Time series methods:

Time series methods use historical information regarding only the dependent variable, based on the assumption that the past pattern will continue into the future.
Methods used are:

•Naive forecasting
•Simple moving averages
•Weighted moving averages
•Exponential smoothing

Naive forecasting

$$F_{t+1} = D_t$$

F_{t+1} = forecast for period $t + 1$
D_t = actual demand in period t

- May take demand trends into account
- May be used to account for seasonal patterns

Advantages: Simplicity and low cost
The method works best when the horizontal trends or seasonal patterns are stable and the random variations are small.

Simple moving average

- Used to estimate the average of a demand time series and thereby remove the effects of random fluctuation.
- Most useful when demand has no pronounced trend or seasonal influences.
- The stability of the demand series generally determines how many periods to include.

Simple moving average

A forecast for period $t + 1$ can be calculated at the end of period t (after the actual demand for period t is known) as

$$F_{t+1} = \frac{\text{Sum of last } n \text{ demands}}{n} = \frac{D_t + D_{t-1} + D_{t-2} + \ldots + D_{t-n+1}}{n}$$

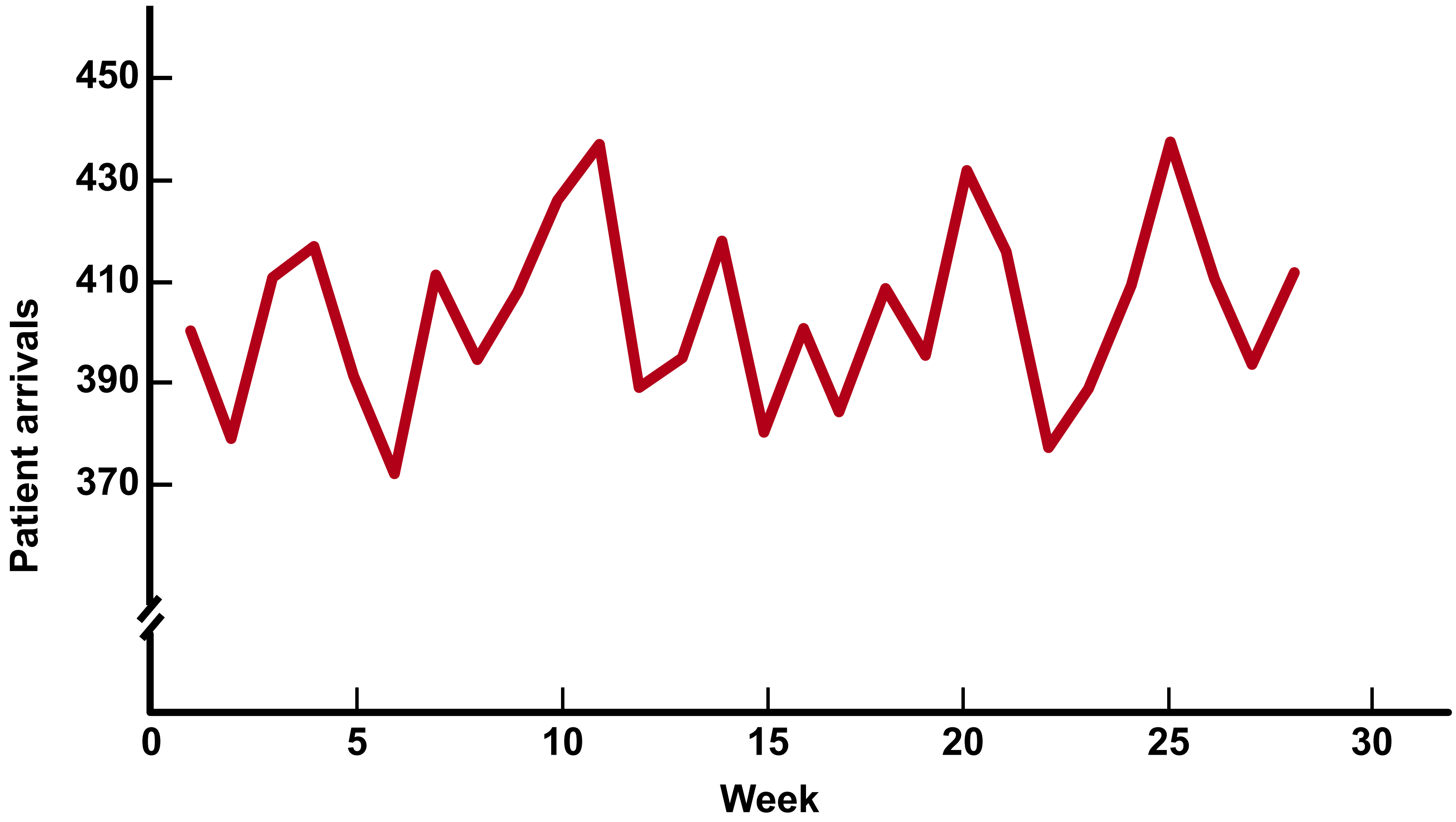
where

D_t = **actual demand in period** t

n = **total number of periods in the average**

F_{t+1} = **forecast for period** $t + 1$

Simple moving average



Simple moving average

EXAMPLE 13.2

a. Compute a three-week moving average forecast for the arrival of medical clinic patients in week 4. The numbers of arrivals for the past three weeks were as follows:

Week	Patient Arrivals
1	400
2	380
3	411

Simple moving average

SOLUTION

a. The moving average forecast at the end of week 3 is

Week	Patient Arrivals
1	400
2	380
3	411

$$F_4 = \frac{411 + 380 + 400}{3} = 397.0$$

b. If the actual number of patient arrivals in week 4 is 415, what is the forecast error for week 4?

c. What is the forecast for week 5?

Simple moving average

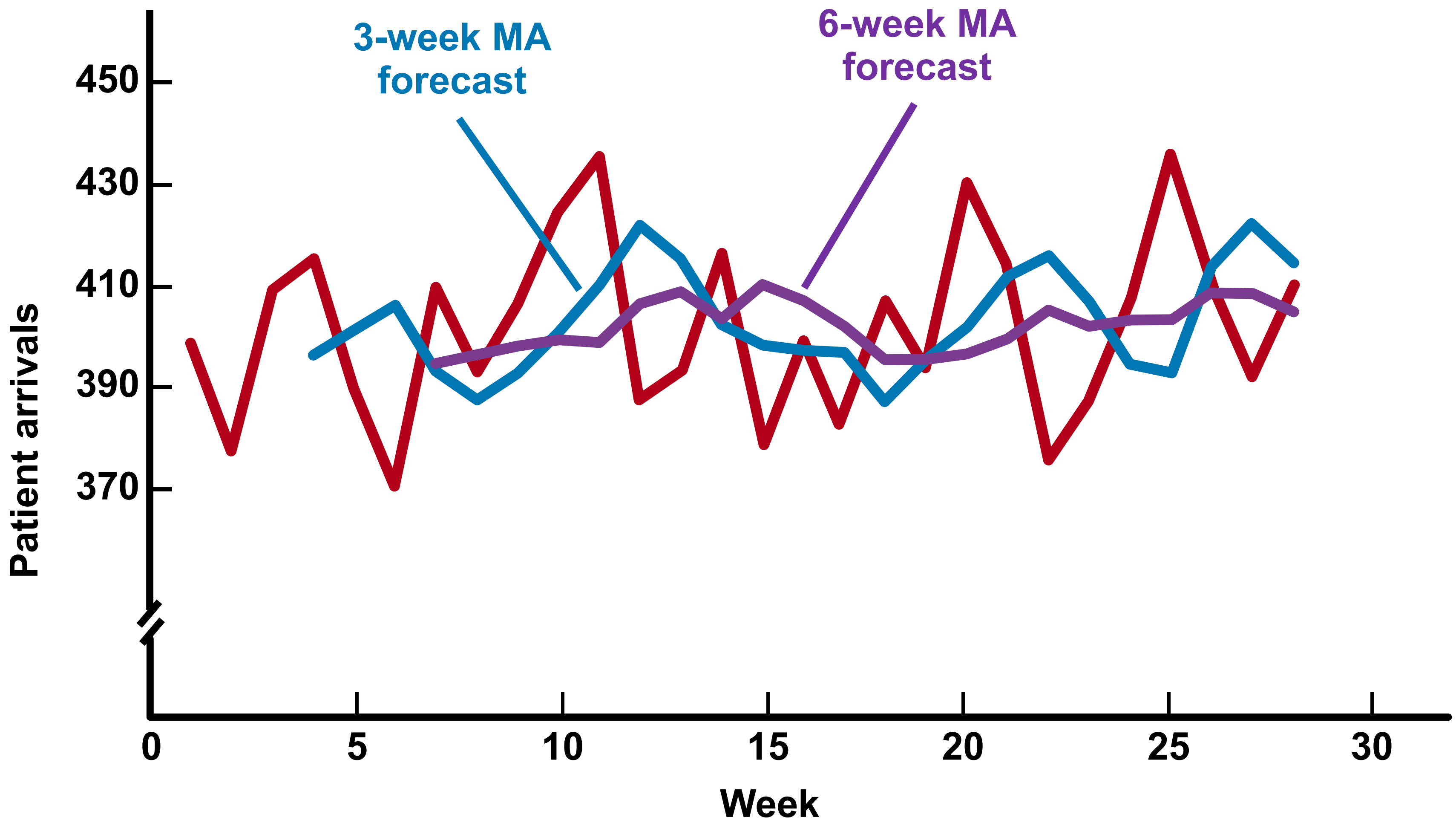
b. The forecast error for week 4 is

$$E_4 = D_4 - F_4 = 415 - 397 = 18$$

c. The forecast for week 5 requires the actual arrivals from weeks 2 through 4, the three most recent weeks of data

$$F_5 = \frac{380 + 411 + 415}{3} = 402.0$$

Simple moving average



Weighted moving average

In the weighted moving average method, each historical demand in the average can have its own weight, provided that the sum of the weights equals 1.0. The average is obtained by multiplying the weight of each period by the actual demand for that period, and then adding the products together:

$$F_{t+1} = W_1D_1 + W_2D_2 + \ldots + W_nD_{t-n+1}$$

A three-period weighted moving average model with the most recent period weight of 0.50, the second most recent weight of 0.30, and the third most recent might be weight of 0.20

$$F_{t+1} = 0.50D_t + 0.30D_{t-1} + 0.20D_{t-2}$$

Weighted moving average

Revisiting the customer arrival data in the earlier example with patient arrivals, let W_1 = 0.50, W_2 = 0.30, and W_3 = 0.20. Use the weighted moving average method to forecast arrivals for month 4.

$$F_4 = W_1D_3 + W_2D_2 + W_3D_1 = 0.50(411) + 0.30(380) + 0.20(400) = 399.5$$

Since we cannot have fractions of people or other discrete items, we round the figure to 400 patients

The actual arrivals in period 4 was, as earlier mentioned, 415 patients. So, what was the forecast error?

$$E_4 = 415 - 400 = 15$$

Weighted moving average

The actual number of arrivals in month 4 is 415. Compute the forecast for month 5

$$F_5 = W_1D_4 + W_2D_3 + W_3D_2 = 0.50(415) + 0.30(411) + 0.20(380) = 406.8$$

Forecast for month 5 is 407customer arrivals

Exponential smoothing

- A sophisticated weighted moving average that calculates the average of a time series by giving recent demands more weight than earlier demands
- Requires only three items of data
  - The forecast for this period
  - The actual demand for this period
  - A smoothing parameter, alpha (α), where $0 \leq \alpha \leq 1.0$
- The equation for the forecast is

$$F_{t+1} = \alpha(\text{Demand this period}) + (1 - \alpha)(\text{Forecast calculated last period})$$

$$F_{t+1} = \alpha D_t + (1 - \alpha)F_t$$ …or the equivalent: $$F_{t+1} = F_t + \alpha(D_t - F_t)$$

Exponential smoothing

- The emphasis given to the most recent demand levels can be adjusted by changing the smoothing parameter
- Larger α values emphasize recent levels of demand and result in forecasts more responsive to changes in the underlying average
- Smaller α values treat past demand more uniformly and result in more stable forecasts
- Exponential smoothing is simple and requires minimal data
- When the underlying average is changing, results will lag actual changes

Exponential smoothing

EXAMPLE 13.3

a. Reconsider the patient arrival data in Example 13.2. It is now the end of week 3. Using $\alpha = 0.10$, calculate the exponential smoothing forecast for week 4.
b. What was the forecast error for week 4 if the actual demand turned out to be 415?
c. What is the forecast for week 5?

Week	Patient Arrivals
1	400
2	380
3	411
4	415

Exponential smoothing

SOLUTION

a. The exponential smoothing method requires an initial forecast. Suppose that we take the demand data for the first two weeks and average them, obtaining (400 + 380)/2 = 390 as an initial forecast. To obtain the forecast for week 4, using exponential smoothing with and the initial forecast of 390, we calculate the average at the end of week 3 as:

$$F_4 = 0.10(411) + 0.90(390) = 392.1$$

Thus, the forecast for week 4 would be 392 patients.

Exponential smoothing

b. The forecast error for week 4 is

$$E_4 = 415 – 392 = 23$$

c. The new forecast for week 5 would be

$$F_5 = 0.10(415) + 0.90(392.1) = 394.4$$

,,,or 394 patients. Note that we used F_4, not the integer-value forecast for week 4, in the computation for F_5. In general, we round off (when it is appropriate) only the final result to maintain as much accuracy as possible in the calculations.

Exponential smoothing with trend

If the historic data shows an obvious trend, we can compensate for that by smoothing, not only the forecast, but also the trend, using an additional smoothing factor, *β.*

So we end up performing three calculations: The average demand, the trend, and the sum of these two, which is the forecast for the coming period.

Exponential smoothing with trend

First we calculate the current estimates of the trend ($A_t - A_{t-1}$).

Estimates for both the average and the trend are smoothed, requiring two constants.

$$\boldsymbol{A_t} = \alpha D_t + (1-\alpha)(A_{t-1} + T_{t-1})$$

$$\boldsymbol{T_t} = \beta(A_t - A_{t-1}) + (1-\beta)\, T_{t-1}$$

$$\boldsymbol{F_{t+1} = A_t + T_t}$$

A_t = *exponential smoothed average of the series in period t*

T_t = *exponential smoothed average of the trend in period t*

α = *smoothing parameter for average (value 0-1)*

β = *smoothing parameter for trend (value 0-1)*

F_{t+1} = forecast for period t+1

Exponential smoothing with trend

In order to plan the operations of a medical lab, the managers request a forecast of the number of patients requesting blood analysis per week. Recent publications about the dammaging effects of cholesterol on the heart has caused a national increase in the request for standard blood tests.

The lab ran an average of 28 blood tests per week during the past four weeks. The trend over that period was 3 additional patients per week. This week's demand was for 27 blood tests. It is decided we should use α=0.20, and β=0.20 to calculate the forecast for next week.

Exponential smoothing with trend

Solution:

A_0=28 patients, and T_0=3 patients
α=0.20, and β=0.20

The forecast for week 2 (next week) is:

A_1=0.20(27) + 0.80(28+3)=30.2

T_1=0.20(30.2 – 28) + 0.80(3)=2.8

F_2=30.2 + 2.8 = 33 patients

If the actual number of blood tests requested in week 2 proved to be 44, the uppdated forecast for week 3 would be:

A_2=0.20(44) + 0.80(30.2+2.8)=35.2

T_1=0.20(35.2 – 30.2) + 0.80(2.8)=3.2

F_3=35.2 + 3.2 = 38.4 (38 patients)

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Multiplicative Seasonal Method

1. For each year, calculate the average demand per season by dividing annual demand by the number of seasons/year. E.g. D=600, and each month is a season: 600/12 = 50 units.
2. For each year, divide actual demand for a season by the average demand per season (seasonal index)
3. Calculate the average seasonal index for each season (add seasonal indicies for a season and divide by the number of years of data.
4. Calculate each season’s forecast for the next year. Begin by estimating average demand/season using naive method, moving average, exponential smoothing, trend adjustment exponential smoothing, etc. Then obtain the seasonal forecast by multiplying the seasonal index by the average demand/season.

Multiplicative Seasonal Method

The manager of Stanley Steemer carpet cleaner company needs a quarterly forecast of the number of customers expected next year. The carpet cleaning business is seasonal, with a peak in the third quarter and a trough in the first quarter. Following are the quarterly demand data from the past four years:

Quarter	Year 1	Year 2	Year 3	Year 4
1	45	70	100	100
2	335	370	585	725
3	520	590	830	1160
4	100	170	285	215
Total:	1000	1200	1800	2200

The manager wants to forecast customer demand for each quarter of year 5, based on the estimate of total year 5 demand of 2600 customers.

Multiplicative Seasonal Method

Solution:

1) Average number of customers/season is:

Y1: 1000/4 = 250

Y2: 1200/4 = 300

Y3: 1800/4 = 450

Y4: 2200/4 = 550

Multiplicative Seasonal Method

2) Seasonal indicies are:

Q	Year 1	Year 2	Year 3	Year 4
1	45/250 = 0.18	70/300 = 0.23	100/450 = 0.22	100/550 = 0.18
2	335/250 = 1.34	370/300 = 1.23	585/450 = 1.30	725/550 = 1.32
3	520/250 = 2.08	590/300 = 1.97	830/450 = 1.84	1160/550 = 2.11
4	100/250 = 0.40	170/300 = 0.57	285/450 = 0.63	215/550 = 0.39

Multiplicative Seasonal Method

3) Average seasonal indecies are:

Q	Average seasonal index
1	(0.18+0.23+0.22+0.18)/4 = 0.20
2	(1.34+1.23+1.30+1.32)/4 = 1.30
3	(2.08+1.97+1.84+2.11)/4 = 2.0
4	(0.40+0.57+0.63+0.39)/4 = 0.5

Multiplicative Seasonal Method

4) Demand for year 5 is total 2600 (2600/4 = 650):

Q	Forecast
1	0.20(650) = 130
2	1.30(650) = 845
3	2.0(650) = 1300
4	0.5(650) = 325

Forecast errors

$E_t = D_t - F_t$

E_t = Forecast error for period t
D_t = Actual demand for period t
F_t = Forecast for period t

Average forecast error

$$\bar{E} = \frac{CFE}{n}$$

CFE = Cumulative sum of Forecast Errors

Mean Square Error (MSE)

$$MSE = \frac{\sum E_t^{\,2}}{n}$$

Forecast errors

Standard deviation σ

$$\sigma = \sqrt{\frac{\sum(E_t - \bar{E})^2}{n-1}}$$

Mean Absolute Deviation (MAD)

$$MAD = \frac{\sum|E_t|}{n}$$

Mean Absolute Percentage Error (MAPE)

$$MAPE = \frac{\sum[|E_t|(100)]/D_t}{n}$$

$$Tracking\ Signal = \frac{CFE}{MAD}$$

Relevant book chapters

- Chapter: “Forecasting demand”
- Exersice: SQ Forecasting

Thank you!

Questions?

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Next session on Tuesday 2018-12-20

Inventory management (pt.1)

Lecturer: Dr. Yuji Yamamoto