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التصميم بمساعدة الحاسب

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التصميم بمســـاعدة الحاســــب

1-Geometry Models.

2-Analysis Models.

3-Sensitivity Analysis.

4-Optimization Design.

1-النماذج الهندسية.

2-تحليل النموذج.

3-حساسية التصميم.

4-التصميم الأمثل.

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Geometry Models:

النماذج الهندسية:

Approximate Geometry:

الهندسة التحليلية:

1-Bezier Model.

1-نموذج Bezier

2-B-Spline Model.

2- نموذج B-Spline

3-NURBS Model.

3- نموذج NURBS

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Bezier Model:

Bezier curve are polynomial curves expressed in terms of Bernstien Polynomails.this model is introduced by paul Bezier.(an engineer at Renault)

نموذج Bezier:

يتألف منحني Bezier مجموعة منحنيات يعبر عنها بتوابع Bernstien .

Principle Form:

$$C(u) = \sum_{i=o}^{m} p_i B_{n.i}(u) \qquad u \in [0,1]$$

Bézier basis functions or Bernstein polynomials :

$$B_{n,i}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$$

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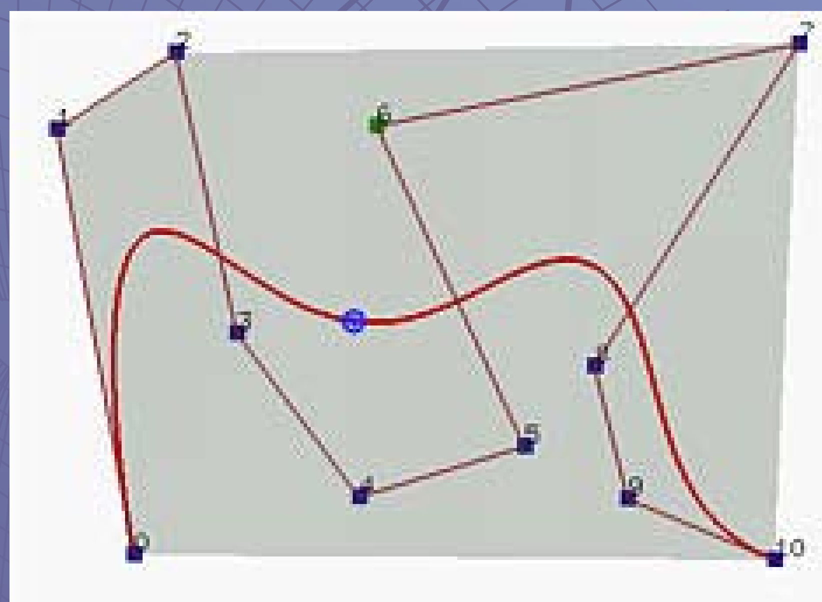
P0**P**1, **P**1**P**2, ..., **P***n*-1**P***n*, called ***legs***, joining in this order form a ***control polyline*** or ***control polygon***



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The important properties of Bezier curve:

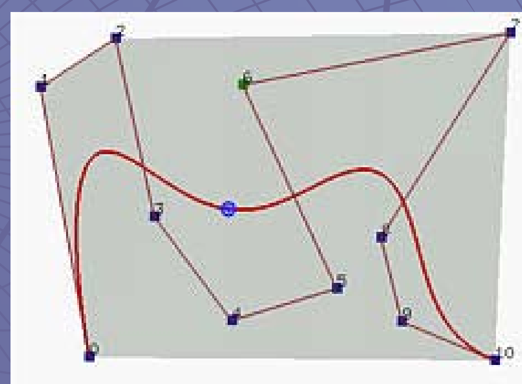
1- The degree of a Bézier curve defined by n+1 control points is n .

2- C(u) passes through P0 and Pn .

3- Non-negativity , All basis functions are non-negative .

4- Partition of Unity , The sum of the basis functions at a fixed u is 1.

5- Convex Hull Property .



6- Changing the position of a control point causes the shape of a Bézier curve to change **globally.**

De Casteljau's algorithm :

De Casteljau's algorithm is used to find a point on Bezier curve according to u , furthermore to subdivide Bezier curve.

$$
\begin{aligned}
&For\ \ j=1\ \ to\ \ j=m\\
&\quad For\ \ i=0\ \ to\ \ i=(m-j)\\
&\quad\quad Compute\ the\ positions\ of\ the\ (m-j+1)\ control\ point\\
&\quad\quad for\ the\ jth\ iteration\ using\ the\ equation:\\
&\quad\quad\quad P_i^{[j]} = P_i^{[j-1]}(1-t) + P_{i+1}^{[j-1]}\,t\\
&\quad End\\
&End
\end{aligned}
$$

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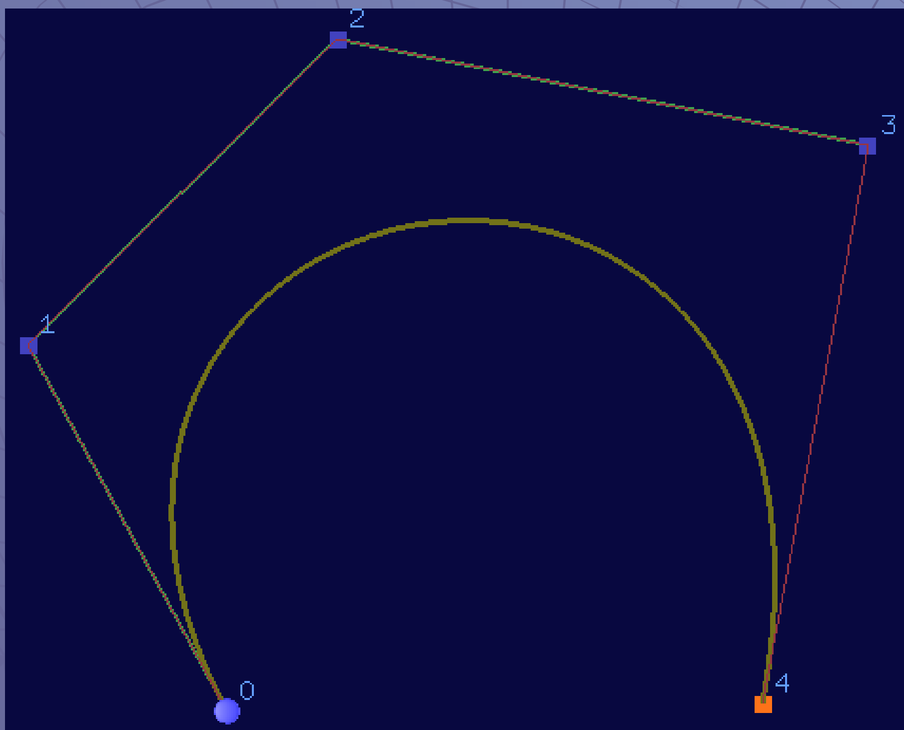

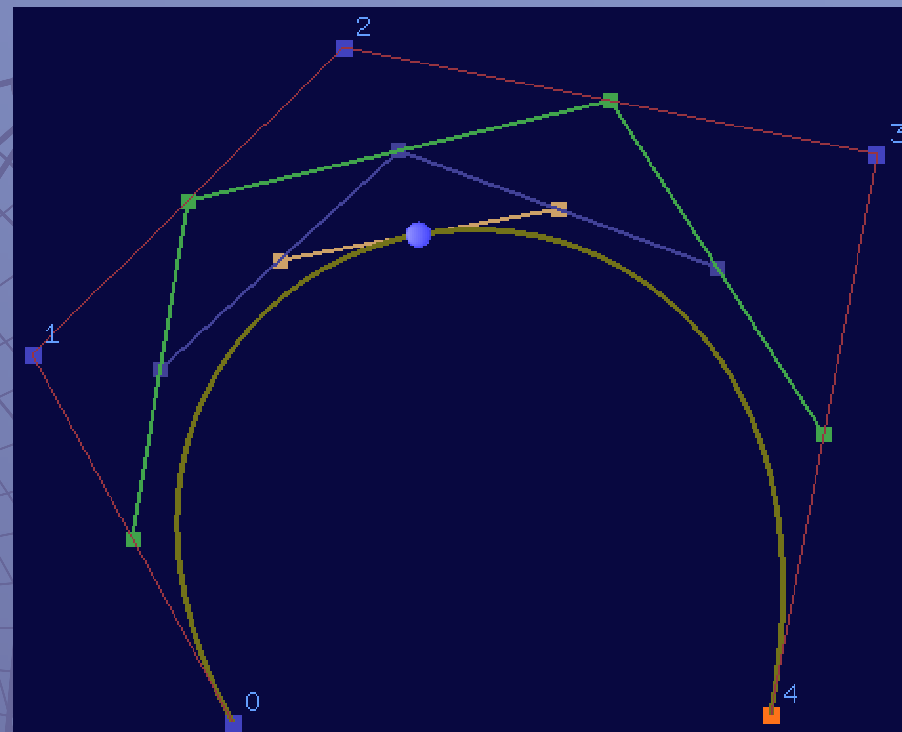

Bezier curve of degree 4

De casteljau algorithm at u=0.5

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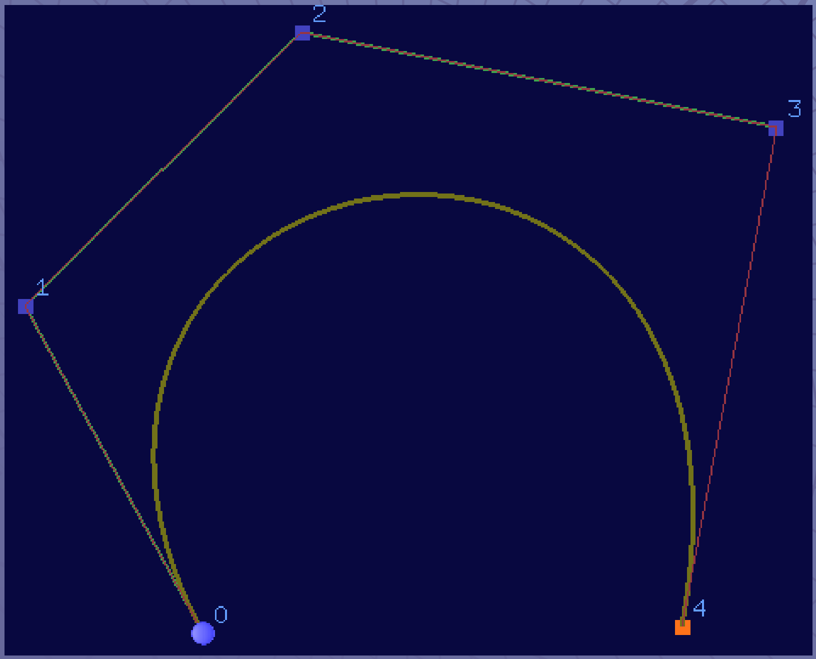
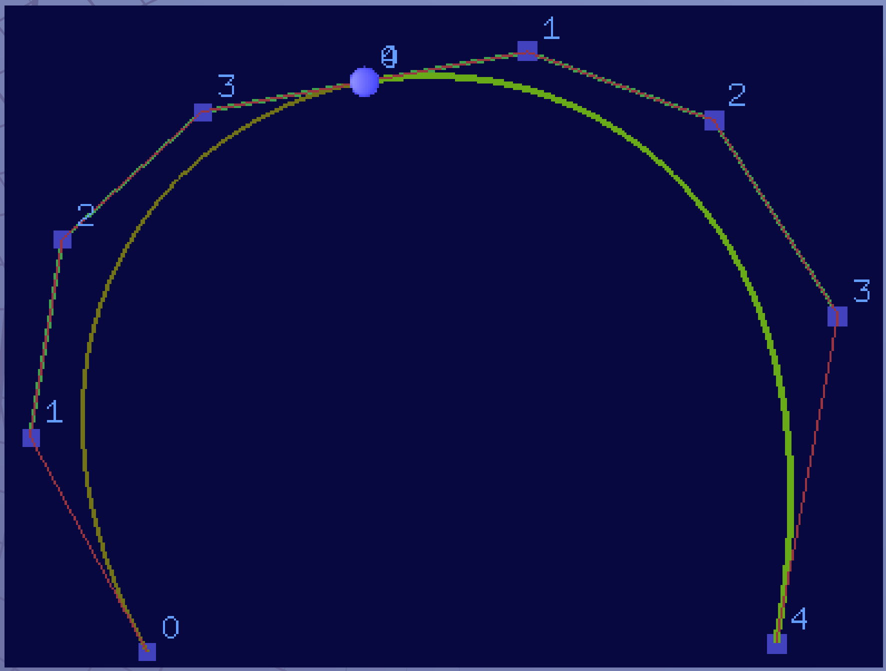
Subdividing a Bézier Curve:

The meaning of *subdividing* a curve is to cut a given Bézier curve at C(*u*) for some *u* into two curve segments by De Casteljau algorithm , each of which is a subset of the original degree n Bézier curve, <u>the resulting Bézier curves must be of degree n</u>.



Bezier curve of degree 4



Subdividing a curve at u=0.5

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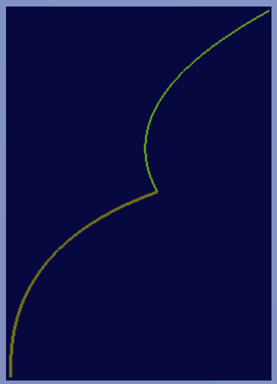
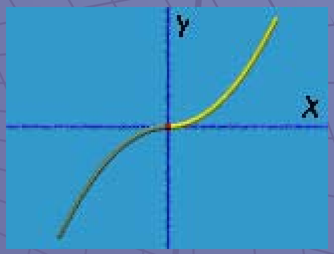
Joining Two Bézier Curves with C1-Continuity:

Continuity Issues:

1- Two curves are C0 continuous at the joining point if f(b) = g(m) .



2- Two curves are C1 continuous at the joining point if the speed (i.e., first derivative) does not change when crossing one curve to the other.



3- two curves are C2 continuous at the joining point if, in addition to the speed, the acceleration (i.e., second derivative) is also the same when crossing one curve to the other..

To achieve C1 continuity at the joining point the ratio of the length of the last leg of the first curve (i.e., |pm - pm-1|) and the length of the first leg of the second curve (i.e., |q1 - q0|) must be n/m.

where:

n : degree of curve D.

m : degree of curve C.

$$C'(1) = D'(0)$$

$$\frac{p_m - p_{m-1}}{Q1 - Q0} = \frac{n}{m}$$

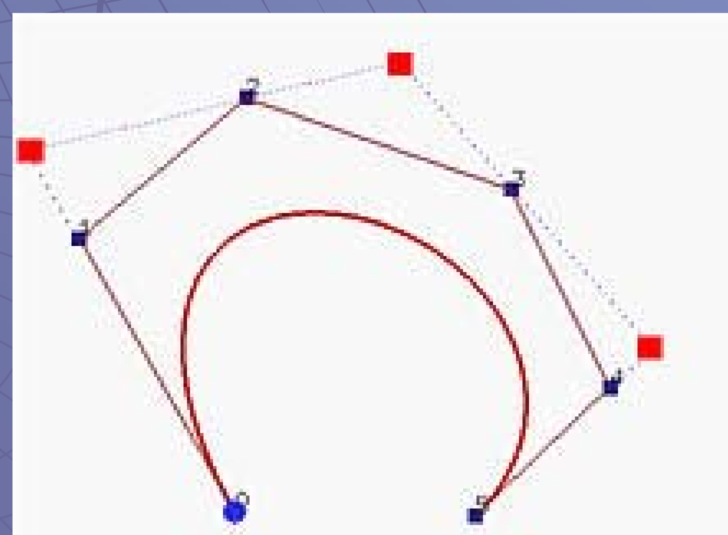
Degree Elevation of a Bézier Curve:

we want to increase the degree of Bezier curve to $n + 1$ *without* changing its shape, so this curve will be defined by $n + 2$ control points.

New control points are computed by the following equation :

$$Q_i = \frac{i}{n+1} p_{i-1} + \left(1 - \frac{i}{n+1}\right) p_i \qquad 1 \le i \le n$$

i	$1 - i/(n+1)$
1	0.8
2	0.6
3	0.4
4	0.2



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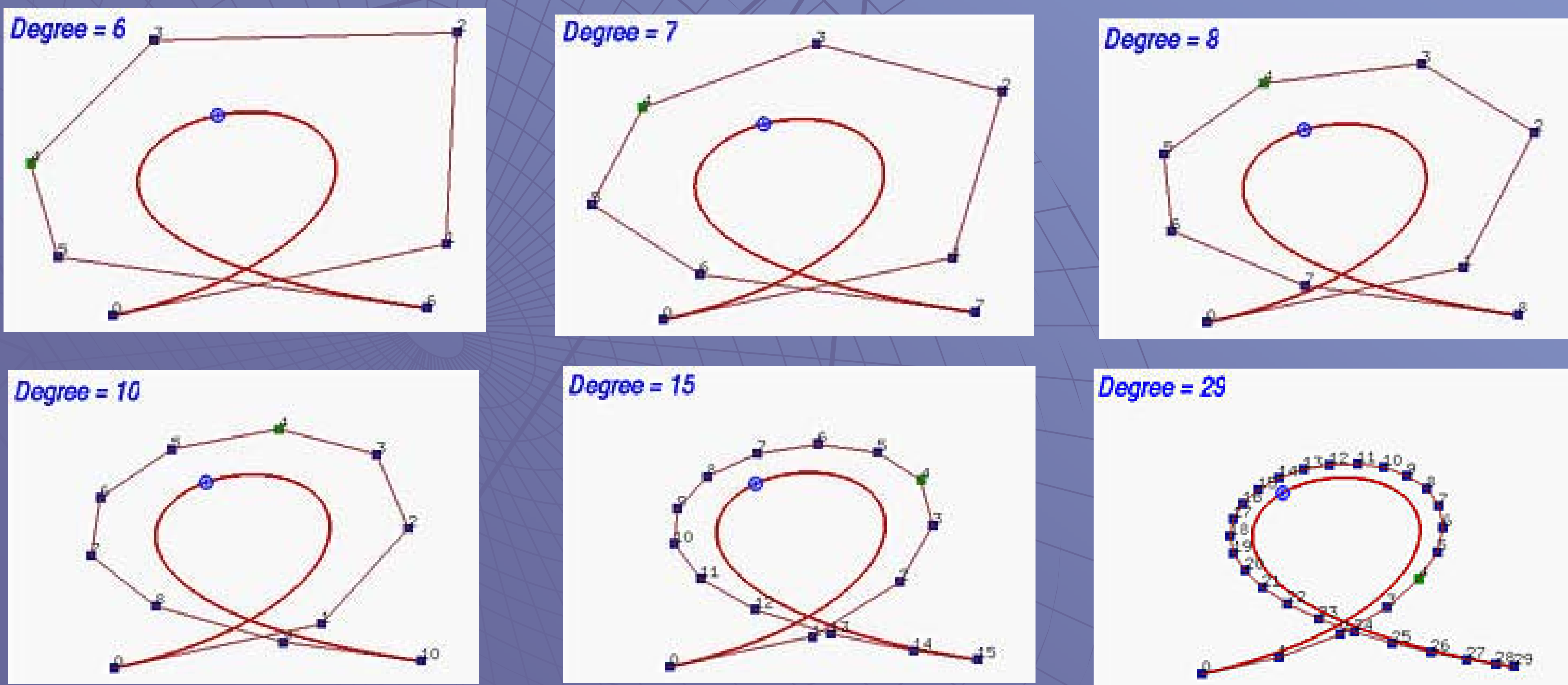
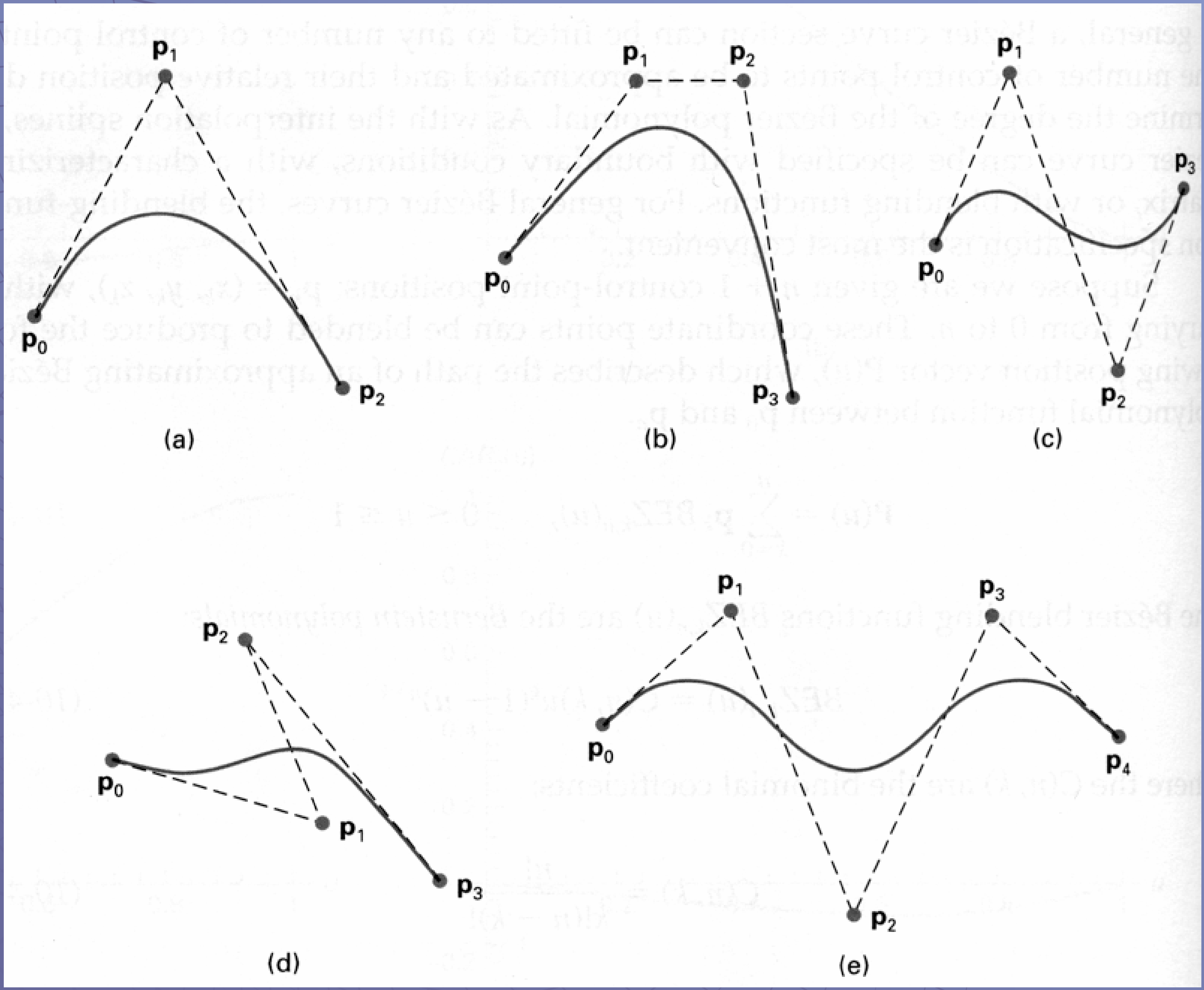
Result:

As the degree increases the number of control points increases and the new control polyline moves toward the curve.



Some Bezier Curves



B-spline:

Definition:

B-Spline curve is a sequence of Bezier curves joining together to form B-Spline curve.

in B-spline curve there are:

1- a set of control points, P0, P1, ..., Pn.

2- a set of knots U = { u0, u1, ..., um }, u0 <= u2 <= u3 <= ... <= um
The ui's are called knots, the set U the knot vector, and the half-open interval [ui, ui+1) the i-th knot span.

If the knots are equally spaced, the knot vector or the knot sequence is said uniform; otherwise, it is non-uniform.

3- a set of basis function these functions are defined as follows:

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$$N_{i,0}(u) = \begin{cases} 1 & \text{if } u_i \le u \le u_{i+1} \\ 0 & \text{otherwise} \end{cases}$$

$$N_{i,p}(u) = \frac{u - u_i}{u_{i+p} - u_i} N_{i,p-1}(u) + \frac{u_{i+p+1} - u}{u_{i+p+1} - u_{i+1}} N_{i+1,p-1}(u)$$

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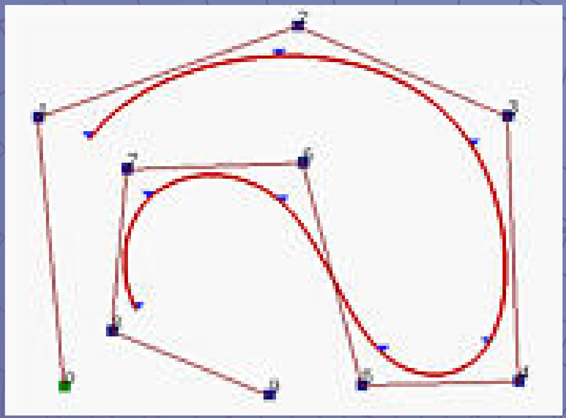
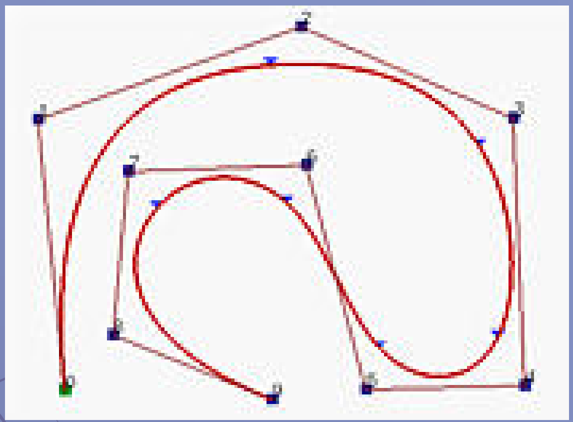
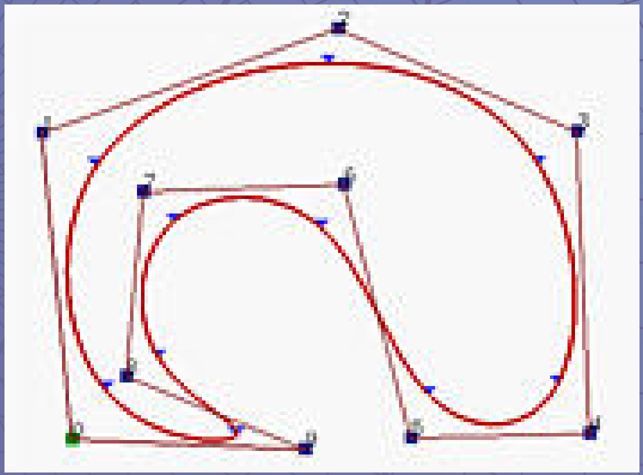
Kinds of B-spline:



open B-spline curve



clamped B-spline curve



closed B-spline curves

To have the clamped effect, the first $p+1 = 4$ and the last 4 knots must be identical.

Important properties of basis function:

1- $N_{i,p}(u)$ is a degree *p* polynomial in *u*

2- $N_{i,p}(u)$ is non-negative.

3- Basis function $N_{i,p}(u)$ is a composite curve.

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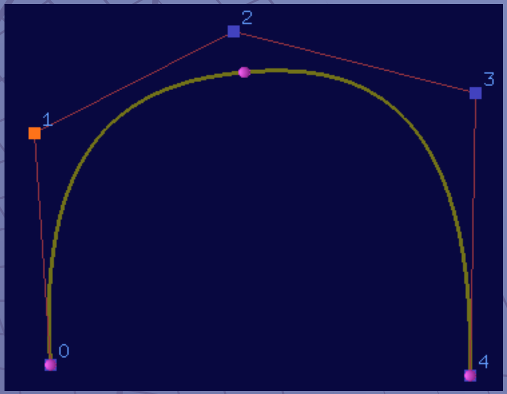
Important properties of B-spline:

1- B-spline curve C(u) is a piecewise curve with each component a curve of degree p.

2- m = n + p + 1 must be satisfied. for example (p=3, n+1=5)

U={0,0,0,0,0.5,1,1,1,1} ; m=8



3- Clamped B-spline curve C(u) passes through the two end control points P0 and Pn.

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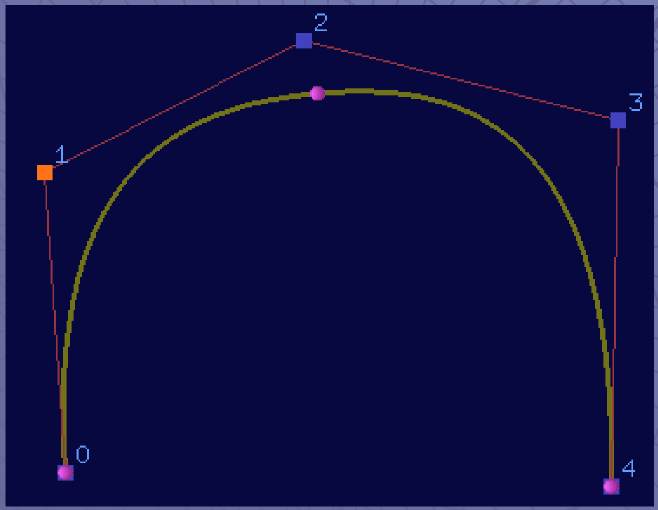
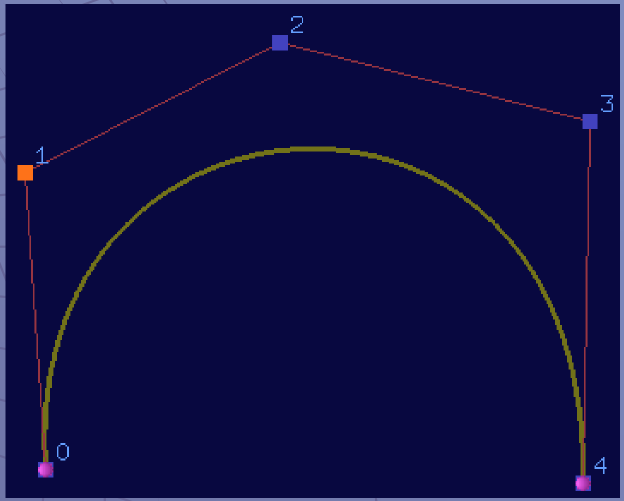
4- Local Modification Scheme: changing the position of control point Pi only affects the curve C(u) on interval [ui, ui+p+1).

5- Bézier Curves Are Special Cases of B-spline Curves. i.e., the degree of a B-spline curve is equal to n, the number of control points minus 1.



P=3



P=4

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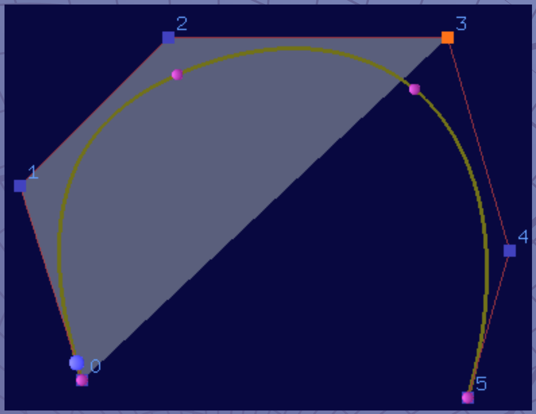
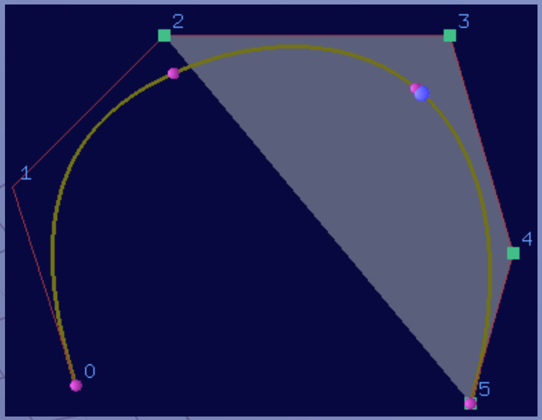
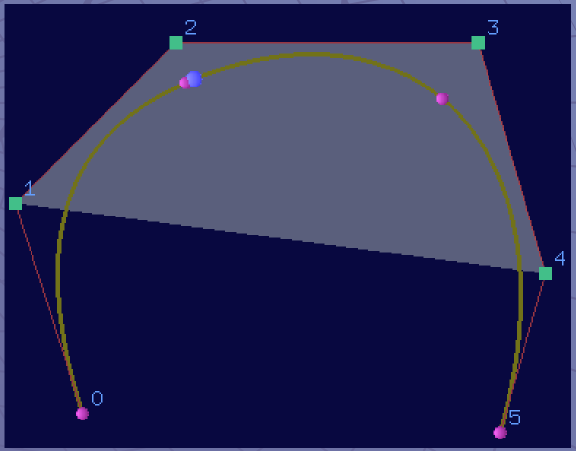
6- Strong Convex Hull Property depends on Bezier segments.







Knot Insertion :

The meaning of ***knot insertion*** is adding a new knot into the existing knot vector **without changing the shape of the curve**. To satisfy the fundamental equality m = n + p + 1

The control points will increase, as well number of Bezier curves will increase.

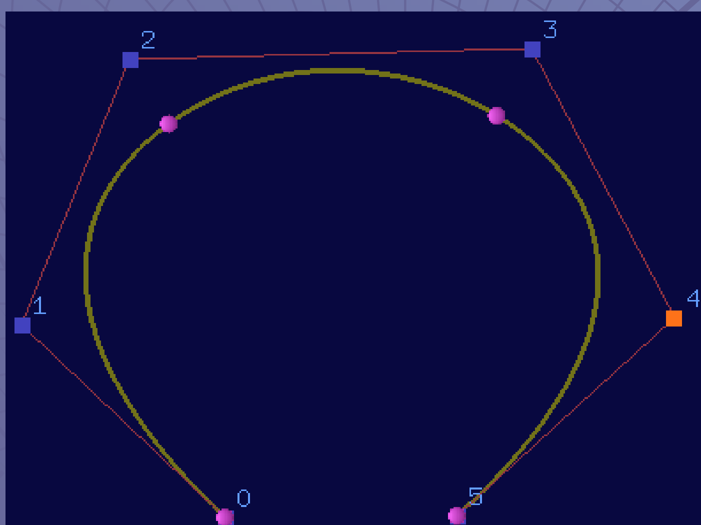
The new set of control points are computed by the following equation:

$$Q_i = (1 - a_i)P_{i-1} + a_i P_i$$

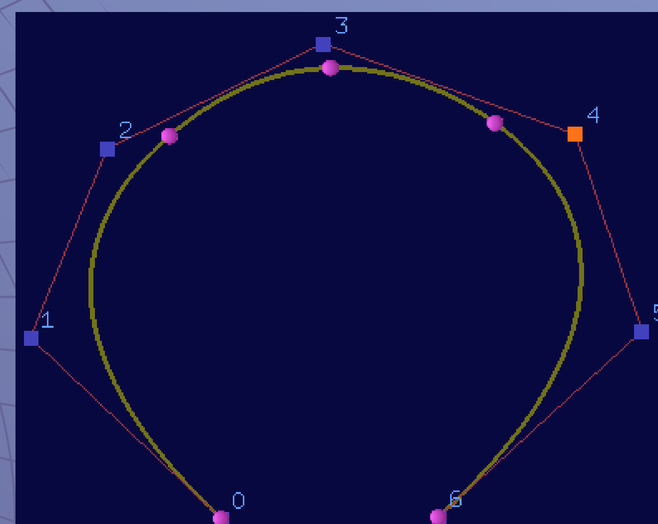
$$a_i = \frac{t - u_i}{u_{i+p} - u_i} \qquad \text{for } k - p + 1 \le i \le k$$

Example about Knot insertion:

B-spline curve of degree 3.



U=0.5



Number of Bezier curve=3

Number of Bezier curve=4

The Advantage of Using B-spline Curves:

First, a B-spline curve can be a Bézier curve.

Second, B-spline curves satisfy all important properties that Bézier curves have.

Third, B-spline curves provide more control flexibility than Bézier curves can do.

For example, the degree of a B-spline curve is separated from the number of control points.

Forth, We can change the position of a control point without globally changing the shape of the whole curve (local modification property).

Fifth, there are other techniques for designing and editing the shape of a curve such as changing knots.

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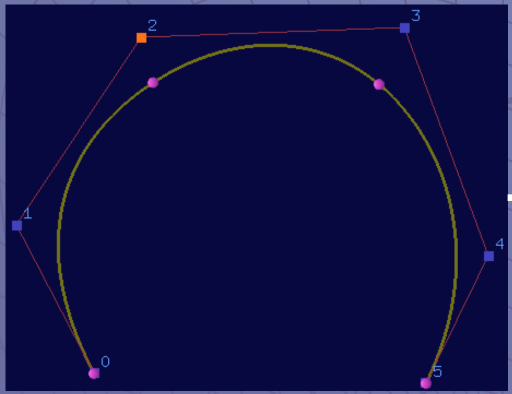
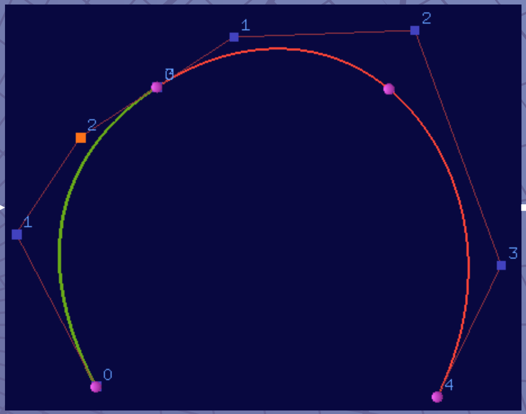
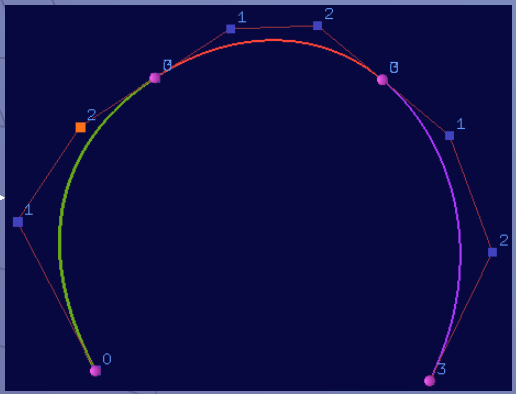
Subdividing a B-spline curve into Bezier curve segments:



Original B-spline



First subdivide



Second subdivide

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Non uniform rational B-spline: (NURBS)

A parametric curve in homogeneous form is referred to as a ***rational curve***.

This kind of curve is defined by the following equation:

$$\mathbf{C}^w(u) = \sum_{i=0}^{n} N_{i,p}(u)\mathbf{P}_i^w = \sum_{i=0}^{n} N_{i,p}(u) \begin{bmatrix} w_i x_i \\ w_i y_i \\ w_i z_i \\ w_i \end{bmatrix} = \begin{bmatrix} \sum_{i=0}^{n} N_{i,p}(u)(w_i x_i) \\ \sum_{i=0}^{n} N_{i,p}(u)(w_i y_i) \\ \sum_{i=0}^{n} N_{i,p}(u)(w_i z_i) \\ \sum_{i=0}^{n} N_{i,p}(u) w_i \end{bmatrix}$$

The point in Homogeneous coordinate has the following form:

$$\mathbf{P}_i^w = \begin{bmatrix} w_i x_i \\ w_i y_i \\ w_i z_i \\ w_i \end{bmatrix}$$

Important properties:

1- If all weights are equal to 1, a NURBS curve reduces to a B-spline curve.

2- NURBS Curves are Rational.

Special case:

If any point has zero weight then it will be ignored.